

Models for neutrino mass with discrete symmetries

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Discrete non-abelian flavor symmetries give in a natural way tri-bimaximal (TBM) mixing as showed in a prototype model. However neutrino mass matrix pattern may be very different from the tri-bimaximal one if small deviations of TBM will be observed. We give the result of a model independent analysis for TBM neutrino mass pattern.

Neutrino data are in well agreement (approximately within 1σ) with the so called tri-bimaximal mixing [1], giving maximal atmospheric angle $\sin^2 \theta_{23} = 1/2$, zero reactor angle $\sin^2 \theta_{13} = 0$ and trimaximal solar angle $\sin^2 \theta_{12} = 1/3$. This is shown in fig.1 where we give the result of the fits of three different groups [2,3,4] represented by three horizontal bands where the blue band is the 3σ region, the red band is the 1σ region, and the green point is the best fit value.

Non-abelian discrete symmetries has been extensively used in order to explain TBM mixing for neutrino, see for instance [5] and reference therein. We provide a simple example based on the discrete abelian flavor symmetry A_4 [6,7,8], namely the even permutations of four objects. This is the smallest finite group with triplet irreducible representation and seems very adequate to accommodate the three flavor families of the Standard Model.

We consider the Altarelli-Feruglio model defined in table 1. The flavon field φ_T is coupled

	L	e^c	μ^c	τ^c	$h_{u,d}$	φ_T	φ_S	ξ
$SU(2)$	2	1	1	1	2	1	1	1
A_4	3	1	1'	1''	1	3	3	1
Z_3^{aux}	ω	ω^2	ω^2	ω^2	1	1	ω	ω

Table 1
Matter assignment of a prototype model for TBM mixing.

only to the charged fermions $Ll^c h \varphi_T$ and φ_S is coupled only to the dimension five operator $LLhh\varphi_S$ in order to preserve the additional Z_3^{aux} . After that the flavon fields take vev, A_4 is spontaneously broken as below

$$\begin{aligned} \langle \varphi_T \rangle &\sim (1, 0, 0) & : & A_4 \rightarrow Z_3; \\ \langle \varphi_S \rangle &\sim (1, 1, 1) & : & A_4 \rightarrow Z_2. \end{aligned} \quad (1)$$

So A_4 is spontaneously broken into Z_3 in the charged fermion sector while into Z_2 in the neutrino sector where Z_2 and Z_3 are two subgroups of A_4 . The misalignment between the two sectors is at the origin of large lepton mixing. In general this is possible thanks to auxiliary abelian symmetries that distinguish the charged and neutral lepton sectors. In the prototype model we are presenting such a auxiliary symmetry is Z_3^{aux} .

It is well know that the alignments $(1, 0, 0)$ or $(1, 1, 1)$ in eq.(1) are natural in A_4 since they leave unbroken the T and the S generators of A_4 respectively. However assuming complex vevs different solutions can be found from the minimization of the potential, see [9,10,11]. In general the solution of the potential containing both the scalar fields φ_S and φ_T is not given by eq.(1) unless terms mixing φ_S and φ_T like $|\varphi_S|^2 |\varphi_T|^2$ are neglected (this is not possible by means of symmetries). It is possible to solve such a problem assuming SUSY [7] or extra-dimension [8].

As a result of the model in table(1) the neutrino mass matrix is $\mu - \tau$ invariant and trimaxi-

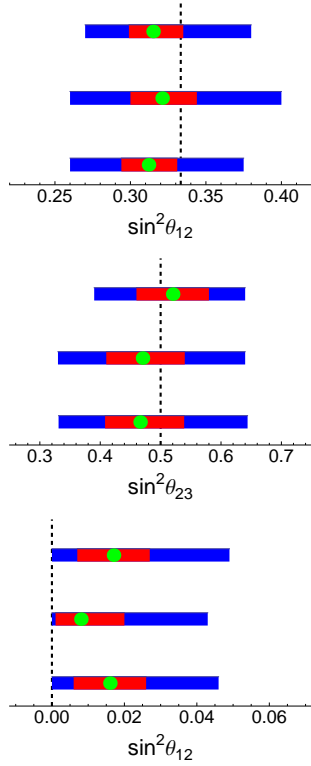


Figure 1. The blue and red ranges are respectively the 3σ and 1σ values for $\sin^2 \theta_{12}$ (up plot), $\sin^2 \theta_{23}$ (middle plot) and $\sin^2 \theta_{13}$ (bottom plot) from the Ref.[2] (up band), [3] (middle band) and [4] (down band) and the green points are the best fit point. The vertical line are the TBM values and the λ_C , λ_C^2 deviations.

mal, that is $m_{\nu_{11}} + m_{\nu_{12}} = m_{\nu_{22}} + m_{\nu_{23}}$ as below

$$m_\nu \equiv \begin{pmatrix} x & y & y \\ y & x+z & y-z \\ y & y-z & x+z \end{pmatrix}. \quad (2)$$

Shortly the neutrino mass matrix has TBM texture. In general, in almost of the models studied in literature, the three angles receive corrections of the same order from next to leading order contributions, that is $\sin^2 \theta_{23} = 1/2 + \mathcal{O}(\epsilon)$, $\sin \theta_{13} = \mathcal{O}(\epsilon)$ and $\sin^2 \theta_{12} = 1/3 + \mathcal{O}(\epsilon)$ where

ϵ is a small perturbation. In order to satisfy solar neutrino data, ϵ can be almost of order λ_C^2 , and the deviation of the reactor angle from TBM must be of the same order. If a larger value for the reactor angle will be measured, currently at 3σ the reactor angle can be as large as λ_C , most of the models will be ruled out.

For small deviation of neutrino mass matrix from TBM pattern, see eq. (2), we expect small deviation of TBM mixing. However assuming small deviations (of order λ_C^2) of the lepton mixing matrix from TBM, the deviations of the neutrino mass matrix from the TBM texture (2) can be very large as indicated in [12]. In order to parameterize the deviation of the neutrino mass matrix from the $\mu - \tau$ exchange symmetry and from the trimaximality, that is $m_{\nu_{11}} + m_{\nu_{12}} = m_{\nu_{22}} + m_{\nu_{23}}$, in [12] the following parameters has been introduced

$$\Delta_e = \frac{m_\nu^{e\mu} - m_\nu^{e\tau}}{m_\nu^{e\mu}}, \quad (3)$$

$$\Delta_{\mu\tau} = \frac{m_\nu^{\mu\mu} - m_\nu^{\tau\tau}}{m_\nu^{\tau\tau}}, \quad (4)$$

$$\Delta_\Sigma = \frac{\Sigma_L - \Sigma_R}{\Sigma_R}, \quad (5)$$

where $\Sigma_L = m_\nu^{ee} + (m_\nu^{e\mu} + m_\nu^{e\tau})/2$ and $\Sigma_R = m_\nu^{\mu\tau} + (m_\nu^{\mu\mu} + m_\nu^{\tau\tau})/2$. The result of the model independent analysis is shown in fig. (2) where a correlation between the parameter Δ_e and $\sin \theta_{13}$ has been given. Such analysis shows that the deviation of the neutrino mass matrix from TBM texture is small, namely of order 10%, if $\sin \theta_{13} < 10^{-3}$ that is very small. When $\sin \theta_{13} \sim \lambda_C^2$, in the sensitivity of future experiments, the deviation of TBM pattern can be very large $\Delta_3 > 1$, see fig. (2).

Also if the model independent analysis do not strictly indicate TBM neutrino pattern, non-abelian discrete symmetries remain a simple and economical description of data. For instance, a model with only A_4 symmetry (no auxiliary symmetries) and without flavons has been proposed in [10] where the standard model has been extended assuming extra Higgs doublets transforming as a triplet of A_4 . Another example of model based on discrete symmetries with interesting phenomenological consequence is given in [13] with approxi-

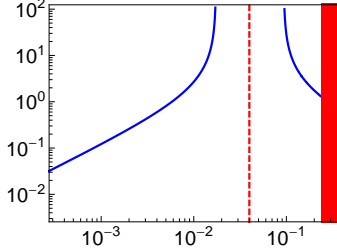


Figure 2. Δ_e vs $\sin \theta_{13}$, the vertical line is the λ_C^2 value.

mate Fritzsch texture for light neutrino mass matrix

$$m_\nu \sim \begin{pmatrix} 0 & b & 0 \\ b & a & c \\ 0 & c & d \end{pmatrix}. \quad (6)$$

The model is based on S_3 flavor symmetry, the permutation of three objects with singlets and doublet irreducible representations. The charged lepton mass matrix is close to diagonal. The two zeros of the Fritzsch texture, give a strong correlation between the reactor angle θ_{13} and the Dirac CP phase δ . In fig. 3 we show such a correlation. The central line is obtained assuming all the observables at the best fit, and we have for maximal CP violation $\delta = \pm\pi/2$ that $\sin \theta_{13}^2 \approx 0.01$ close to the indication of ref. [14].

Acknowledgments

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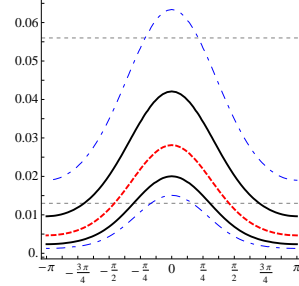


Figure 3. $\sin^2 \theta_{13}$ vs the Dirac CP phase δ . The central line is for the best fit, while the other lines are the one and three σ .

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